JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 34, No. 1, January–February 1997

Exact Design of Partial Coverage Satellite Constellations over Oblate Earth

Der-Ming Ma* and Wen-Chiang Hsu†
Tamkang University, Tamsui, Taiwan 25137, Republic of China

The developed algorithm of computational satellite visibility periods over an oblate Earth is used to construct the time line. Based on the time line the design of the partial coverage satellite constellation can be obtained exactly. The criterion for choosing the best constellation is to minimize the number of satellites with the smallest value of inclination to meet the gap requirement. The method constructs the time lines exactly and then obtains the precise satellite constellations for the time gap requirements. The developed C++ computer program can find the most efficient arrangement of the satellites in constellations and the optimal value of inclination in less than 10 min on a Pentium-based personal computer.

Nomenclature

a_0 = semimajor axis
$E_{\min}$ = minimum elevation angle required to see the satellite
F = visibility function
f = Earth flattening factor
G_1, G_2 = constants defined in Eq. (6)
h = observer's height
i = inclination
M_0 = initial mean anomaly
$(M_0)_n$ = initial mean anomaly of nth satellite
N_{pass} = number of passes over the observer of a satellite
N_{sat} = number of satellites
$\bar{n}$ = orbit mean motion
R = equatorial radius of Earth
$\boldsymbol{r}$ = geocentric position vector of the satellite
r_e = Earth radius at ground point
$\boldsymbol{r}_{\text{obs}}$ = geocentric position vector of the observer
$T_{G_{\text{des}}}$ = gap time requirement
T_{G_j} = jth invisible time interval
$T_{G_{\max}}$ = largest gap on the time line
$T_{G_{\text{2nd}}}$ = second gap on the time line
T_n = first rise time of the nth satellite
T_{rep} = repeating time
T_{set_j} = jth set time of satellite
t = time
$\hat{z}$ = geodetic zenith unit vector
β_1' = angle defined in Eq. (7)
β_2' = angle defined in Eq. (8)
λ = observer's longitude
ν = mean anomaly
$\hat{\rho}$ = unit vector pointing from the observer to the satellite
$\boldsymbol{\rho}$ = slant range vector from the observer to the satellite
τ = orbit period
ϕ = observer's geodetic latitude
ϕ' = observer's geocentric latitude
Ω = ascending node
$(\Omega_0)_n$ = initial ascending node of nth satellite
$\dot{\Omega}$ = rate of ascending node
ω_e = Earth rotating rate

Presented as Paper 94-3721 at the AIAA/AAS Astrodynamics Conference, Scottsdale, AZ, Aug. 1–3, 1994; received June 20, 1995; revision received Sept. 10, 1996; accepted for publication Sept. 17, 1996.

*Associate Professor, Department of Aerospace Engineering. Member AIAA.

†Graduate Student, Department of Mechanical Engineering; currently Research Assistant, Department of Aerospace Engineering.

Introduction

WITH the development of lighter, cheaper satellites and the possibility of smaller, relatively inexpensive launch systems, the low-altitude satellite constellations receive increased attention. Therefore the design of satellite constellations providing partial coverage and global coverage are becoming more important. Partial coverage typically means coverage of certain regions of the Earth, with gap times in coverage no longer than some specified maximum. The purpose of this study is to develop an exact procedure necessary for designing the best partial coverage constellation over an oblate Earth.

Numerous researchers have studied satellite constellation design from the point of view of continuous coverage for global coverage[1–5] and coverage of certain regions.[6] Several research works on noncontinuous coverage of a small region have been published.[7–10] In Ref. 7, the constellations with circular orbits, and having repeating ground tracks on Earth, are shown to give better partial coverage over those with nonrepeating ground tracks. It shows how to pick the best ascending node location and initial mean anomaly for each satellite and how to design the best constellation by directly computing the coverage of the satellite covering a ground point.

In this paper, instead of obtaining the time line (the time the satellite will see the ground point) by the simple coverage model, we use the computation algorithm of satellite visibility periods over an oblate Earth developed in Ref. 11. The design procedure from Ref. 7 (referred to as the HET design procedure) is modified to obtain the best constellation. The criterion for choosing the best constellation is to minimize the number of satellites for meeting the gap requirements with the smallest value of inclination.

In Ref. 7 it was shown that the satellite constellations having circular orbit and repeating ground track over the Earth have better partial coverage. In this paper the satellite constellations with repeating ground tracks and circular orbits are used.

Starting with the construction of a satellite time line by solving the satellite visibility function obtained from Ref. 11, the paper discusses the algorithm of satellite constellation design based on the time line meshing. Several numerical examples are then given to illustrate the algorithm.

Satellite's Visibility Problem

The function for the satellite visibility problem is given by[11]

$$F = \sin^{-1}(\hat{\rho} \cdot \hat{z}) - E_{\min} \tag{1}$$

It is trivial that when the satellite is visible, $F > 0$, so that when F changes sign from negative to positive the satellite is rising. A set is oppositely characterized by F's changing sign from positive to negative. It is obvious that the time line can be constructed by solving Eq. (1). In Eq. (1) $\hat{z}$ is given by

$$\hat{z} = \begin{pmatrix} \cos\phi\cos\lambda \\ \cos\phi\sin\lambda \\ \sin\phi \end{pmatrix} \tag{2}$$

The slant range vector $\boldsymbol{\rho}$ is given by the following vector relationship:

$$\boldsymbol{\rho} = \boldsymbol{r} - \boldsymbol{r}_{\mathrm{obs}} \tag{3}$$

The geocentric position vector of the satellite, assuming the satellite is in a circular orbit, is

$$\boldsymbol{r} = a_0 \begin{pmatrix} \cos\Omega\cos\nu - \sin\Omega\sin\nu\cos i \\ \sin\Omega\cos\nu - \cos\Omega\sin\nu\cos i \\ \sin\nu\sin i \end{pmatrix} \tag{4}$$

where

$$\nu = M_0 + \bar{n}t, \qquad \Omega = \Omega_0 + \dot{\Omega}t$$

The observer's position vector can be computed using the following expression:

$$\boldsymbol{r}_{\mathrm{obs}} = \begin{pmatrix} -G_1\cos\phi\cos\lambda \\ -G_1\cos\phi\sin\lambda \\ -G_2\sin\phi \end{pmatrix} \tag{5}$$

where the constants G_1 and G_2 are given as

$$G_1 = \frac{R}{\sqrt{1-(2f-f^2)\sin\phi}} + h$$
$$G_2 = \frac{R(1-f)^2}{\sqrt{1-(2f-f^2)\sin\phi}} + h \tag{6}$$

The values of the parameters R and f of the oblate Earth are taken from the WGS84 model. Figure 1 is a typical plot of the visibility function of a satellite over a particular ground point. Every peak on the curve represents the maximum elevation angle of the satellite. When the values of the peaks are positive the satellite is visible, and when the values of the peaks are negative, the satellite is invisible. Along the time axis, the visibility intervals can be obtained. Referring to the time line obtained from Fig. 1, we can obtain $T_{G_{\max}}$ and $T_{G_{2\mathrm{nd}}}$ of a single satellite. The $T_{G_{\max}}$ is the gap between the last set time and the first rise time of the satellite in the period of T_{rep} and is the largest invisible period of the satellite. The $T_{G_{2\mathrm{nd}}}$ is the second largest invisible period of the satellite. Those quantities are important in designing the constellation. Based on the algorithm developed in Ref. 11, it is possible to solve the time line from the visibility function. Since the characteristics of the visibility function depend on ϕ and i, to speed up the process, a special procedure is devised. Figure 2 is the flow diagram of the procedure. In the procedure, two iteration loops are involved. The outer iteration loop is to obtain the local maximum value of the visibility function and the corresponding time. When the region of the peak value is approximately located, the derivative-free minimization algorithm as suggested in Ref. 11 is used. The inner iteration loop solves for the rise/set times of the satellite. The proposed algorithm of finding the rise/set time is efficient and accurate.

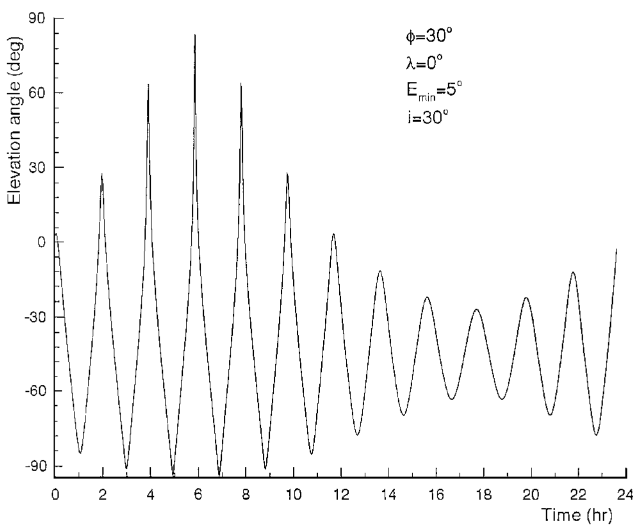

Fig. 1 Visibility function of a satellite.

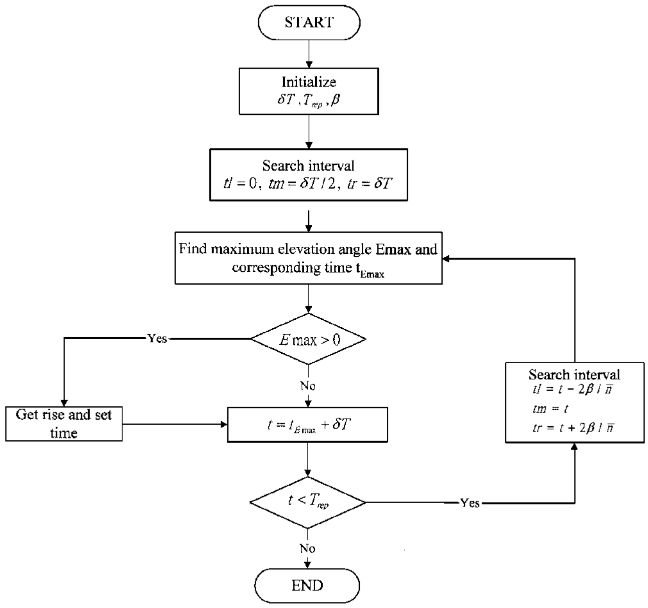

Fig. 2 Flowchart of solving the time line.

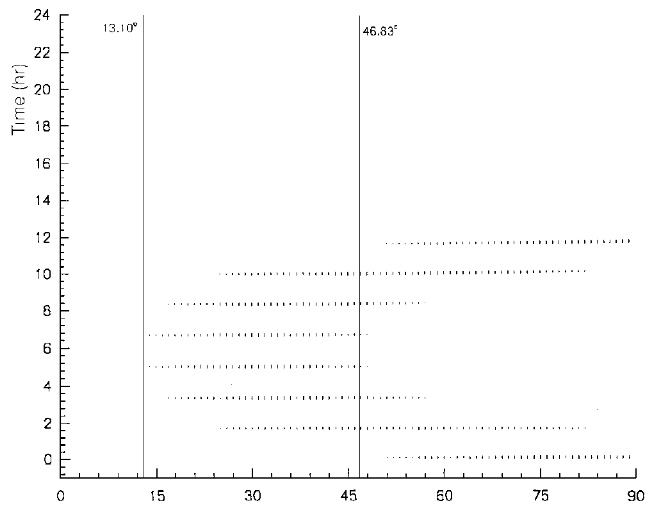

Fig. 3 Time lines of different inclinations.

Figure 3 is the plot of the time lines of a satellite over the ground point for different inclination angles. It is obvious that the visibility period varies widely as a function of orbit inclination. Observing the visibility periods, three conclusions can be made (refer to Fig. 4).

1) There are no visibility periods for the inclinations below $\phi' - \beta_1'$, where β_1' is given as

$$\beta_1' = \pi/2 - \Delta\phi - E_{\min} - \sin^{-1}[(r_e/a_0)\sin(\pi/2 + \Delta\phi + E_{\min})] \tag{7}$$

In the equation, $\Delta\phi = \phi - \phi'$.

2) The time lines are sets of line segments, when the inclinations are between $\phi' - \beta_1'$ and $\phi' + \beta_2'$, where β_2' is given as

$$\beta_2' = \pi/2 + \Delta\phi - E_{\min} - \sin^{-1}[(r_e/a_0)\sin(\pi/2 - \Delta\phi + E_{\min})] \tag{8}$$

3) The time lines are unions of two sets of line segments, when the inclinations are above $\phi' + \beta_2'$. Therefore the inclinations of the

Table 1 Constellations for λ = 0 deg, ϕ = 30 deg, and $E_{\min}$ = 5 deg

		Exact design							Walker's design				
Alt., km	Max gap req., h	N_{sat}	i, deg	Gap, h	Ascending node, deg	Initial mean anomalies, deg	Meshing type	Computer run time, min	N_{sat}	i, deg	Gap, h	$T/P/F$	First, asc. node, deg $M_0 = 0$
Near 500 ($N = 15$)	0.1	19	41	0.076	348, 25, 61, 98, 134, 171, 207, 244, 280, 317, 354, 30, 67, 103, 140, 176, 213, 24, 286	0, 172, 343, 155, 327, 138, 310, 121, 283, 105, 276, 88, 260, 71, 243, 55, 226, 38, 209	2	5.23	N/A	N/A	N/A	N/A	N/A
	0.5	7	35	0.47	348, 47, 106, 166, 225, 284, 343	0, 192, 25, 217, 50, 242, 75	2	3.5	7	38	0.5	7/7/4	10
	1	4	35	0.82	348, 78, 168, 258	0, 90, 180, 271	2	1.52	4	35	0.82	4/4/1	12
	2	2	35	1.79	348, 168	0, 180	1	1.47	2	35	1.79	2/2/1	12
	4	2	25	3.42	24, 204	0, 180	1	1.12	2	28	3.38	2/2/1	0
	6	2	20	5.05	276, 96	0, 180	1	1.35	2	28	3.38	2/2/1	0
	8	2	17	6.7	24, 204	0, 180	1	1.33	2	28	3.38	2/2/1	0
	12	1	52	11.99	216	0	0	1.15	1	52	12	N/A	0
	18	1	20	16.78	276	0	0	0.18	1	28	15.1	N/A	0
	24	1	14	21.73	24	0	0	0.05	1	28	15.1	N/A	0
Near 800 ($N = 14$)	0.1	15	39	0.088	302, 40, 138, 235, 333, 71, 169, 268, 4, 102, 200, 298, 36, 133, 231	0, 71, 142, 213, 283, 354, 65, 136, 207, 278, 349, 59, 130, 201, 272	2	3.76	N/A	N/A	N/A	N/A	N/A
	0.5	5	49	0.5	212, 140, 68, 356, 284	0, 287, 215, 142, 70	2	1.78	5	50	0.5	5/5/1	6
	1	4	28	0.82	302, 39, 135, 232	0, 88, 178, 267	2	1.5	4	29	1	4/2/2	6
	2	2	28	1.67	302, 122	0, 0	1	1.4	2	28	1.67	2/2/0	19
	4	2	17	2.77	212, 32	0, 360	1	1	2	28	1.67	2/2/0	19
	6	2	12	4.54	302, 122	0, 360	1	0.85	2	28	1.67	2/2/0	19
	8	2	10	6.31	212, 32	0, 0	1	0.8	2	28	1.67	2/2/0	19
	12	1	48	11.15	212	0	0	1.17	1	49	11.14	N/A	6
	18	1	12	16.3	302	0	0	0.15	1	28	12.79	N/A	19
	24	1	8	19.9	302	0	0	0.05	1	28	12.79	N/A	19
Near 1200 ($N = 13$)	0.1	13	48	0.097	0, 155, 310, 104, 259, 54, 207, 3, 158, 313, 108, 262 ,57	0, 148, 296, 84, 232, 20, 168, 316, 104, 252, 40, 188, 336	2	4.76	N/A	N/A	N/A	N/A	N/A
	0.5	5	50	0.49	180,102,24,307,229	0,291,222,153,85	2	2.57	5	55	0.5	5/5/1	0
	1	4	20	0.88	166,241,316,31	0,103,206,309	2	1.62	4	28	0.8	4/2/2	0
	2	2	8	1.97	346,166	0,180	1	0.85	2	28	1.74	2/2/0	0
	4	2	5	3.9	166,346	0,180	1	0.78	2	28	1.74	2/2/0	0
	6	2	3	5.9	346,166	0,180	1	0.72	2	28	1.74	2/2/0	0
	8	2	3	5.9	346,166	0,180	1	0.75	2	28	1.74	2/2/0	0
	12	1	20	11.89	166	0	0	0.45	1	28	11.86	N/A	0
	18	1	3	17.69	346	0	0	0.017	1	28	11.86	N/A	0
	24	1	3	17.69	346	0	0	0.017	1	28	11.86	N/A	0

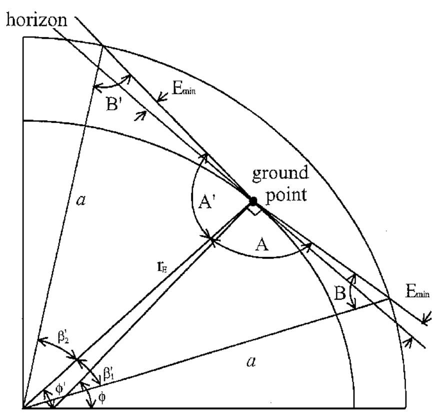

Fig. 4 Visible and invisible regions.

visible constellations for ground points of latitude ϕ are in the range between $\phi' - \beta_1'$ and 90 deg.

Satellite Constellation Design

It is assumed that every satellite in the constellation has the same characteristics except the ascending node and the mean anomaly. They are related by the following equations[7]:

$$(\Omega_0)_n = \mathrm{MOD}[(\Omega_0)_1 + (T_n - T_1)(\omega_e - \dot{\Omega}),\ 2\pi]$$
$$(M_0)_n = \mathrm{MOD}(\{2\pi - \mathrm{MOD}[(T_n - T_1),\ \tau]\bar{n}\},\ 2\pi) \tag{9}$$

It is important to obtain the ascending node, which is the key to maximizing the number of passes of the satellite over the desired ground point. Therefore the key to design of constellations with many satellites is judicious choice of relative satellite time positions. Optimal constellations are obtained by placing individual satellites along the time in the best fashion.

In the case where the gap time requirement is larger than the largest gap between passes on adjacent revolutions of a single satellite, only one satellite is required.

Example 1

Let the ground point be $\lambda = 0$ deg, $\phi = 30$ deg, and $E_{\min} = 5$ deg, if $T_{G_{\mathrm{des}}}$ is 18 h. For the constellation near 500 km, $T_{G_{\mathrm{des}}} > T_{G_{\max}} = 16.7785$ h. Then only one satellite is needed (refer to Table 1).

If the gap time requirement is smaller than the largest gap between passes on adjacent revolutions of a single satellite, time lines from two or more satellites must be meshed together. Depending on the relationships between $T_{G_{\max}}$, $T_{G_{2\mathrm{nd}}}$, and $T_{G_{\mathrm{des}}}$, there are two kinds of meshing algorithms for the satellites to be used: tandem meshing and intersected meshing.

Tandem meshing: $T_{G_{\max}} > T_{G_{\mathrm{des}}} > T_{G_{2\mathrm{nd}}}$. Suppose we have the constellation of N_{sat} satellites that satisfies the time gap requirement. Each satellite has a period of $T_{\mathrm{rep}} - T_{G_{\max}}$ that satisfies the mission requirement. Then the constellation has the period of $N_{\mathrm{sat}}(T_{\mathrm{rep}} - T_{G_{\max}})$, which satisfies the requirement. Let the gap between each satellite be equal or less than or equal to the value of $T_{G_{\mathrm{des}}}$, or

$$T_{G_{\mathrm{des}}} \le \frac{\left[T_{\mathrm{rep}} - N_{\mathrm{sat}}\left(T_{\mathrm{rep}} - T_{G_{\max}}\right)\right]}{N_{\mathrm{sat}}} \tag{10}$$

Table 2 Constellations for $\lambda = 0$ deg, $\phi = 30$ deg, and $E_{\min} = 30$ deg

		Exact design							Walker's design				
Alt., km	Max gap req., h	N_{sat}	i, deg	Gap, h	Ascending node, deg	Initial mean anomalies, deg	Meshing type	Computer run time, min	N_{sat}	i, deg	Gap, h	$T/P/F$	First, asc. node, deg $M_0 = 0$
Near 500 ($N = 15$)	0.1	42	34	0.095	12, 22, 32, 41, 51, 61, 71, 80, 90, 100, 110, 20, 129, 139, 149, 159, 168, 179, 188, 198, 208, 217, 227, 237, 247, 256, 266, 276, 286, 296, 305, 315, 325, 335, 345, 354, 4, 14, 24, 33, 43, 53	0, 213, 67, 280, 133, 347, 200, 53, 266, 120, 333, 186, 40, 253, 106, 320, 173, 26, 239, 93, 306, 159, 13, 226, 79, 293, 146, 359, 212, 66, 279, 132, 346, 199, 52, 266, 119, 332, 185, 39, 252, 105	2	7.4	N/A	N/A	N/A	N/A	N/A
	0.5	12	34	0.43	12, 70, 128, 186, 244, 301, 359, 57, 115, 173, 231, 289	0, 212, 63, 275, 126, 338, 189, 41, 253, 104, 316, 167	2	0.88	11	34	0.45	11/11/17	12
	1	6	34	0.84	12, 76, 140, 205, 269, 333	0, 117, 235, 652, 110, 227	2	0.68	7	33	0.94	7/7/2	0
	2	3	34	1.62	12, 132, 252	0, 0, 0	1	1.12	3	34	1.62	3/3/0	12
	4,	3	29	2.83	28848, 168	0, 0, 0	1	0.9	3	29	2.84	3/3/0	0
	6	2	34	5.1	12, 192	0, 180	1	1.08	2	34	5.91	2/2/0	12
	8	2	29	6.75	288, 108	0, 180	1	0.9	2	29	7.56	2/2/0	0
	12	2	24	11.72	276, 96	0, 180	1	0.75	2	28	9.2	2/2/0	20
	18	1	34	16.86	12	0	0	0.28	1	34	16.86	N/A	12
	24	1	24	23.45	276	0	0	0.017	1	28	20.13	N/A	8
Near 800 ($N = 14$)	0.1	29	36	0.09	122, 159, 196, 234, 271, 308, 345, 26, 60, 97, 135, 171, 208, 246, 283, 321, 358, 35, 72, 109, 147, 184, 221, 258, 296, 333, 10, 47, 85	0, 199, 37, 236, 74, 273, 112, 311, 149, 348, 187, 26, 224, 63, 262, 100, 299, 138, 336, 175, 14, 212, 51, 250, 88, 287, 126, 325, 163	2	5.77	N/A	N/A	N/A	N/A	N/A
	0.5	10	33	0.5	122, 158, 194, 230, 266, 302, 338, 14, 50, 86	0, 216, 72, 288, 144, 0, 216, 72, 289, 145	2	0.87	10	34	0.5	10/10/6	19
	1	6	31	0.88	302,11,80,149,218,287	0, 144, 228, 342, 196, 210	2	0.88	6	32	0.83	6/3/2	19
	2	3	31	1.73	302, 62, 182	0, 120, 240	2	1	3	32	1.72	3/3/1	19
	4	3	26	2.44	212, 332, 92	0, 120, 240	1	0.85	3	28	3.01	3/3/0	2
	6	2	31	4.61	302, 122	0, 360	1	1	2	32	4.59	2/2/0	19
	8	2	26	6.37	212, 32	0, 360	1	0.83	2	28	6.35	2/2/0	6
	12	2	21	9.95	212, 32,	0, 360	1	0.7	2	28	6.35	2/2/0	6
	18	1	31	16.39	302	0	0	0.23	1	32	16.38	N/A	19
	24	1	21	21.71	212	0	0	0.03	1	28	18.12	N/A	6
Near 1200 ($N = 13$)	0.1	25	40	0.09	346, 60, 135, 209, 283, 357, 72, 146, 221, 295, 9, 84, 158, 232, 307, 21, 96, 170, 244, 319, 33, 107, 182, 256, 331	0, 113, 226, 339, 92, 205, 318, 71, 184, 297, 50, 163, 277, 30, 143, 256, 9, 122, 235, 348, 101, 214, 327, 80, 194	2	6	N/A	N/A	N/A	N/A	N/A
	0.5	9	44	0.43	180, 216, 253, 290, 327, 3, 40, 77, 114	0, 242, 124, 7, 249, 131, 14, 256, 138	2	1.31	10	34	0.46	10/10/2	7
	1	5	31	0.99	166, 238, 310, 22, 94,	0, 144, 288, 72, 216	2	1.08	5	32	0.99	5/5/2	0
	2	3	23	1.99	346, 106, 226	0, 240, 120	1	0.82	3	28	1.94	3/3/2	13
	4	2	30	3.99	346, 166	0, 180	1	1	2	30	4	2/2/1	0
	6	2	23	5.93	346, 166	0, 180	1	0.9	2	28	5.87	2/2/1	13
	8	2	19	5.93	346, 166	0, 180	1	0.7	2	28	5.87	2/2/0	13
	12	1	84	10.38	14	0	0	1.92	1	84	10.38	N/A	14
	18	1	23	17.73	346	0	0	0.13	1	28	17.68	N/A	13
	24	1	19	19.69	166	0	0	0.02	1	28	17.68	N/A	13

Explicitly, the number of satellites in the constellation is

$$N_{\text{sat}} = \text{INT}\left(\frac{T_{\text{rep}}}{T_{\text{rep}} - T_{G_{\max}} + T_{G_{\text{des}}}}\right) + 1 \tag{11}$$

The time shift between the satellites in the constellation is

$$\Delta T = T_{\text{rep}}/N_{\text{sat}} \tag{12}$$

and

$$T_n = T_1 + (n-1)\Delta T, \qquad \text{for} \qquad n = 1, \ldots, N_{\text{sat}} \tag{13}$$

In summary, the algorithm is to place the second satellite after the last pass of the first satellite by the amount of ΔT and the third satellite after the second by the same amount. The procedure is repeated until the number of the satellites equals N_{sat}.

Example 2

Let the ground point be $\lambda = 0$ deg, $\phi = 30$ deg, and $E_{\min} = 30$ deg, if $T_{G_{\text{des}}}$ is 12 h. For the constellation near 500 km, $T_{G_{\max}} = 20.1275\,\text{h} > T_{G_{\text{des}}} > T_{G_{2\text{nd}}} = 1.6175$ h. Then the tandem meshing algorithm is used. In the example, $N_{\text{sat}} = 2$ and $\Delta T = 11.74$ h (refer to Table 2).

Intersected meshing: $T_{G_{\text{des}}}$ is less than all of the gaps except $T_{G_{\max}}$.

To have the more efficient arrangement, we maximize the time gap of the meshed time line by the Brent method,[12] and the time shift is obtained from the interval

$$T_{\text{set}_j} - T_1 \leq \Delta T \leq T_{\text{set}_j} - T_1 + T_{G_j} \qquad j = 1, \ldots, N_{\text{pass}} \tag{14}$$

In summary, the procedure starts with the number of the satellites equal to three and the time shift rule given by Eq. (14). The time shifts

Table 3 Constellations for λ = 0 deg, ϕ = 50 deg, and E_{min} = 5 deg

		Exact design							Walker's design				
Alt., km	Max gap req., h	N_{sat}	i, deg	Gap, h	Ascending node, deg	Initial mean anomalies, deg	Meshing type	Computer run time, min	N_{sat}	i, deg	Gap, h	$T/P/F$	First, asc. node, deg $M_0 = 0$
Near 500 ($N = 15$)	0.1	18	57	0.093	348, 9, 31, 52, 73, 94, 116, 137, 158, 180, 201, 222, 243, 265, 286, 307, 329, 350	0, 41, 81, 122, 163, 204, 244, 285, 326, 6, 47, 88, 128, 169, 210, 251, 291, 332	2	4.87	N/A	N/A	N/A	N/A	N/A
	0.5	7	55	0.48	84, 142, 201, 259, 317, 16, 74	0, 204, 49, 253, 98, 302, 146	2	2.05 (17.1)[a]	6	64	0.47	6/2/1	0
	1	4	55	0.92	84, 174, 264, 354	0, 89, 178, 268	2	1.93 (2.6)	4	55	0.92	4/4/1	12
	2	2	55	1.98	84, 264	0, 180	1	1.76	2	55	1.99	2/2/1	12
	4	2	48	3.57	216, 36	0, 180	1	1.55	2	48	3.58	2/2/1	0
	6	2	42	5.19	204, 24	0, 180	1	1.32	2	42	5.98	2/2/0	0
	8	2	38	6.81	288, 108	0, 180	1	1.15	2	38	7.61	2/2/0	0
	12	1	74	10.7	12	0	0	1.53	1	74	10.71	N/A	12
	18	1	42	16.97	204	0	0	0.32	1	42	16.98	N/A	12
	24	1	33	23.48	276	0	0	0.017 (0.2)	1	33	23.49	N/A	0
Near 800 ($N = 14$)	0.1	15	63	0.089	212, 118, 23, 289, 195, 100, 6, 272, 178, 83, 349, 255, 160, 66, 332	0, 240, 121, 1, 241, 122, 2, 243, 123, 3, 244, 124, 4, 245, 125	2	5.43	N/A	N/A	N/A	N/A	N/A
	0.5	5	73	0.47	302, 14, 86, 158, 230	0, 72, 144, 217, 289	2	2.4	5	73	0.47	5/5/1	19
	1	3	73	0.89	302, 62, 182	0, 120, 240	2	2.15	3	73	0.9	3/3/1	19
	2	2	52	1.61	302, 122	0, 0	1	1.63	2	52	1.62	2/2/0	19
	4	2	43	2.97	212, 32	0, 0	1	1.33	2	43	2.98	2/2/0	6
	6	2	37	4.69	122, 302	0, 0	1	1.15	2	37	4.7	2/2/0	19
	8	2	32	6.46	32, 212	0, 360	1	0.98	2	33	6.43	2/2/0	7
	12	1	62	11.41	212	0	0	1.17	1	62	11.42	N/A	6
	18	1	37	16.48	122	0	0	0.27	1	37	16.49	N/A	19
	24	1	27	23.53	302	0	0	0.03	1	28	21.74	N/A	5
Near 1200 ($N = 13$)	0.1	11	60	0.1	0, 93, 186, 279, 12, 105, 198, 291, 24, 117, 210	0, 231, 103, 334, 205, 77, 308, 179, 51, 282, 153,	2	4.52	N/A	N/A	N/A	N/A	N/A
	0.5	5	71	0.5	180, 103, 25, 308, 230	0, 287, 214, 141, 68	2	2.38	5	72	0.5	5/5/3	25
	1	3	73	0.92	0, 100, 201	0, 134, 267	2	2.22	3	73	0.95	3/3/1	0
	2	2	50	1.72	0, 180	0, 180	1	1.58	2	50	1.73	2/2/0	0
	4	2	40	2.24	346, 166	0, 180	1	1.28	2	40	3.13	2/ 2/0	0
	6	2	28	5.99	346, 166	0, 180	1	0.92	2	28	6	2/2/1	13
	8	1	84	6.94	180	0	0	1.85	1	84	6.94	N/A	14
	12	1	61	10.49	180	0	0	1.13	1	62	10.46	N/A	14
	18	1	28	17.8	346	0	0	0.15	1	28	17.81	N/A	13
	24	1	22	23.6	166	0	0	0.017	1	28	17.81	N/A	13

[a]The value in parentheses is the computer run time from Ref. 7 (SUN workstation).

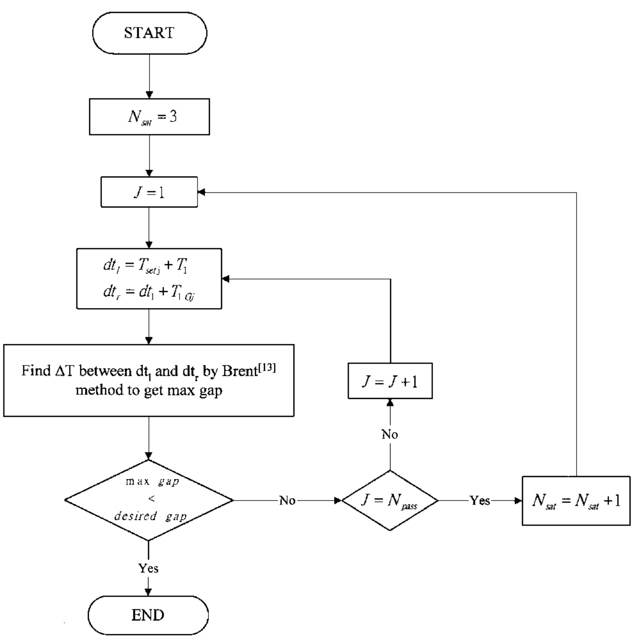

Fig. 5 Flowchart of intersection meshing scheme.

are the same for all satellites in the constellation. If the time gap required is not satisfied, one more satellite will be added. The procedure continues until the time gap required is fulfilled. Figure 5 illustrates the flow diagram of intersection meshing. The intersected meshing algorithm can be used for the zero time gap requirement.

Example 3

Let the ground point be $\lambda = 0$ deg, $\phi = 30$ deg, and $E_{\min} = 30$ deg, if $T_{G_{des}}$ is 1 h. For the constellation near 500 km, $T_{G\max} = 16.8486\,\text{h} > T_{G_{2nd}} = 1.61867\,\text{h} > T_{G_{des}}$. Then the intersection meshing algorithm is used. In the example, $\Delta T = 4.19402$ h (refer to Table 2).With the scheme discussed earlier, the flow diagram of satellite constellation design is shown in Fig. 6. The criterion for choosing the best constellation is to minimize the number of satellites with the smallest value of inclination to meet the gap requirements. The design procedure starts with the inclination equals to $\phi' - \beta_1'$ and ends at 90 deg. The set of the minimum number of satellites with the smallest value of inclination from the design procedure is the optimum set.

Numerical Results

Tables 1–4 show the results obtained from the new design algorithm. Table 5 shows the design results for high $E_{\min}$ and short time gap requirements. In the table the meshing scheme used (1 for tandem meshing, 2 for intersection meshing, and 0 when

Table 4 Constellations for $\lambda = 0$ deg, $\phi = 50$ deg, and $E_{\min} = 30$ deg

		Exact design							Walker's design				
Alt., km	Max gap req., h	N_{sat}	i, deg	Gap, h	Ascending node, deg	Initial mean anomalies, deg	Meshing type	Computer run time, min	N_{sat}	i, deg	Gap, h	$T/P/F$	First, asc. node, deg $M_0 = 0$
Near 500 ($N = 15$)	0.1	41	55	0.095	348, 51, 113, 176, 239, 301, 4, 67, 129, 192, 255, 317, 20, 83, 145, 208, 271, 334, 36, 99, 162, 224, 287, 350, 52, 115, 178, 240, 303, 6, 68, 131, 194, 256, 319, 22, 84, 147, 210, 272, 335	0, 140, 280, 60, 199, 339, 119, 259, 39, 179, 318, 98, 238, 18, 158, 298, 77, 217, 357, 137, 277, 57, 196, 336, 116, 256, 36, 176, 315, 95, 235, 15, 155, 295, 74, 214, 354, 134, 274, 54, 193	2	10.88	N/A	N/A	N/A	N/A	N/A
	0.5	11	54	0.48	12, 45, 78, 110, 143, 176, 209, 241, 274, 307, 340	0, 229, 97, 326, 195, 63, 292, 161, 29, 258, 126	2	1.23 (5.3)[a]	11	55	0.48	11/11/7	12
	1	6	54	0.95	12, 72, 132, 192, 252, 312	0, 180, 1, 181, 2, 182	2	1.05 (0.8)	6	55	0.95	6/6/3	12
	2	3	54	1.6	12, 132, 252	0, 360, 360	2	1.68	3	55	1.6	3/3/0	12
	4	3	50	2.92	24, 144, 264	0, 0, 0	1	1.47	3	50	2.92	3/3/0	0
	6	2	54	5.24	12, 192	0, 180	1	1.63	2	55	5.23	2/2/1	12
	8	2	50	6.85	24, 204	0, 180	1	1.47	2	50	7.66	2/2/0	0
	12	2	44	10.13	288, 108	0, 180	1	1.18	2	44	11.77	2/2/1	8
	18	1	54	17.06	12	0	0	0.47	1	55	17.06	N/A	11
	24	1	44	21.92	288	0	0	0.05 (0.2)	1	44	23.55	N/A	7
Near 800 ($N = 14$)	0.1	33	56	0.097	328, 339, 351, 2, 13, 24, 36, 47, 58, 69, 81, 92, 103, 114, 126, 137, 148, 159, 171, 182, 193, 205, 216, 227, 238, 250, 261, 272, 283, 295, 306, 317, 328	0, 202, 45, 247, 89, 292, 134, 336, 179, 21, 223, 66, 268, 110, 313, 155, 357, 200, 42, 244, 87, 289, 131, 334, 176, 18, 221, 63, 265, 108, 310, 182, 355	2	6.33	N/A	N/A	N/A	N/A	N/A
	0.5	10	60	0.42	32, 132, 232, 332, 72, 172, 272, 12, 112, 212	0, 40, 80, 120, 160, 200, 240, 280, 319, 359	2	1.55	11	56	0.5	11/11/7	10
	1	5	60	0.86	32, 99, 165, 232, 299	0, 147, 293, 80, 227	2	1.62	6	52	0.84	6/3/2	19
	2	3	52	1.7	328, 88, 208	0, 120, 240	2	1.55	3	52	1.71	3/3/1	19
	4	2	60	3.01	32, 212	0, 0	1	1.87	2	60	3.01	3/3/1	6
	6	2	52	4.75	328, 148	0, 0	1	1.55	2	52	4.76	2/2/0	19
	8	2	47	6.49	32, 212	0, 5	1	1.33	2	47	6.49	2/2/0	6
	12	1	80	11.51	32	0	0	1.62	1	80	11.51	N/A	6
	18	1	52	16.59	328	0	0	0.48	1	52	16.6	N/A	19
	24	1	40	23.59	302	0	0	0.033	1	43	21.82	N/A	6
Near 1200 ($N = 13$)	0.1	26	56	0.088	0, 107, 215, 322, 70, 177, 285, 32, 140, 247, 354, 102, 209, 317, 64, 172, 279, 26, 134, 241, 349, 96, 204, 311, 59, 166	0, 43, 87, 130, 173, 217, 260, 303, 346, 30, 73, 116, 160, 203, 246, 290, 333, 16, 60, 103, 146, 189, 233, 276, 319, 3	2	4.8	N/A	N/A	N/A	N/A	N/A
	0.5	10	51	0.48	0, 108, 216, 324, 72, 180, 288, 35, 143, 251	0, 37, 74, 111, 148, 185, 222, 259, 297, 334	2	1.57	9	63	0.48	9/9/2	0
	1	5	50	0.92	0, 72, 144, 216, 288	0, 144, 288, 82, 217	2	1.52	5	51	0.9	5/5/2	0
	2	3	50	1.83	0, 120, 240	0, 240, 120	2	1.42	3	51	1.81	3/3/0	0
	4	2	59	2.32	346, 166	0, 180	1	1.72	2	59	3.22	2/2/0	0
	6	2	50	4.21	0, 180	0, 180	1	1.4	2	51	5.09	2/2/0	0
	8	2	40	7.97	0, 180	0, 180	0	1.05	2	40	7.98	2/2/0	0
	12	1	81	10.59	14	0	0	1.58	1	81	10.59	N/A	14
	18	1	44	17.92	346	0	0	0.23	1	44	17.93	N/A	14
	24	1	37	23.61	166	0	0	0.017	1	37	23.63	N/A	0

[a]The value in parentheses is the computer run time from Ref. 7 (SUN workstation).

only one satellite is required) and computation times used on a Pentium-based personal computer are also listed for reference. Compared with Walker's results from Ref. 7, the new design yields equal or better results for most cases evaluated. But for several cases of short time gap requirements, there are one or two satellite numbers more than the results from Ref. 7. After carefully reviewing the time line of the Ref. 7 results, we find the new design gives the exact results. For example, for the case of $\phi = 30$ deg, $E_{\min} = 30$ deg, and $T_{G_{\rm des}} = 0.5$ h, the result from Ref. 7 gives eight satellites with the ascending nodes 346.29, 113.29, 151.30, 158.96, 166.62, 293.61, 331.62, and 339.28; the initial anomalies 0, 151.98, 18.76, 279.40, 180, 331.98, 198.76, and 99.38; and the inclination of 40 deg. With the data of the aforementioned constellation, the time line is reconstructed, which gives $T_{G_{\max}} = 1.81869$ h and places behind pass 21; and $T_{G_{\rm 2nd}} = 1.49893$ h and places behind pass 41. From Table 1, the new design gives nine satellites at the inclination of 44 deg and the $T_{G_{\max}} = 0.43$ h. Because of the exact model being used, it does provide accurate results. In Tables 3 and 4, the computer run times (on a SUN workstation) from Ref. 7 are also shown.

Table 5 Constellations for $\lambda = 0$ deg and $E_{\min} = 60$ deg

			Exact design							Walker's design				
Alt., km	ϕ, deg	Max gap req., h	N_{sat}	i, deg	Gap, h	Ascending node, deg	Initial mean anomalies, deg	Meshing type	Computer run time, min	N_{sat}	i, deg	Gap, h	$T/P/F$	First, asc. node, deg $M_0 = 0$
Near 800 ($N = 14$)	30	0.5	16	33	0.48	32, 98, 164, 231, 297, 3, 69, 135, 202, 268, 334, 40, 106, 173, 239, 305	0, 153, 306, 100, 253, 46, 199, 352, 145, 299, 92, 245, 38, 191, 344, 138	2	0.7	16	34	0.5	16/16/2	6
		1	8	33	0.91	32, 55, 77, 100, 122, 145, 167, 190	0, 45, 90, 135, 179, 224, 269, 314	2	0.63	8	33	0.95	8/4/2	0
		2	4	33	1.77	32, 325, 259, 192	0, 214, 68, 282	2	0.96	4	33	1.84	4/4/3	0
	50	0.5	18	50	0.43	304, 316, 327, 339, 350, 2, 14, 25, 37, 49, 60, 72, 83, 95, 107, 118, 130, 142	0, 197, 35, 232, 69, 267, 104, 301, 138, 336, 173, 10, 208, 45, 242, 80, 277, 114	2	0.96	17	54	0.36	17/17/3	5
		1	10	50	0.86	304, 326, 347, 9, 30, 52, 73, 95, 117, 138	0, 58, 116, 174, 232, 290, 349, 47, 105, 163	2	0.85	9	50	0.89	9/9/4	19
		2	5	50	1.74	304, 344, 24, 64, 104,	0, 159, 318, 118	2	0.81	4	54	1.75	4/4/2	6
Near 1200 ($N = 13$)	30	0.5	15	34	0.46	346, 5, 23, 42, 60, 79, 97, 116, 135, 153, 172, 190, 209, 227, 246	0, 119, 237, 356, 114, 233, 351, 110, 238, 347, 105, 224, 342, 101, 220	2	0.83	14	33	0.48	14/7/2	0
		1	8	28	0.98	166, 192, 218, 243, 269, 294, 321, 346	0, 25, 50, 76, 101, 126, 151, 176	2	0.73	8	28	0.98	8/8/3	0
		2	4	29	1.98	166, 225, 284, 344	0, 310, 260, 210, 277	2	0.7	3	36	2	3/3/2	14
	50	0.5	15	55	0.49	318, 331, 345, 358, 11, 25, 38, 51, 65, 78, 91, 105, 118, 131, 145	0, 187, 13, 200, 27, 213, 40, 227, 53, 240, 66, 253, 80, 266, 93	2	1.28	14	53	0.5	14/7/2	0
		1	8	53	1	318, 342, 6, 30, 54, 78, 102, 126	0, 48, 96, 144, 192, 240, 288, 337	2	1.2	8	52	1	8/8/3	0
		2	4	53	1.88	318, 16, 74, 132	0, 324, 288, 253	2	1.16	4	53	1.88	4/4/1	0

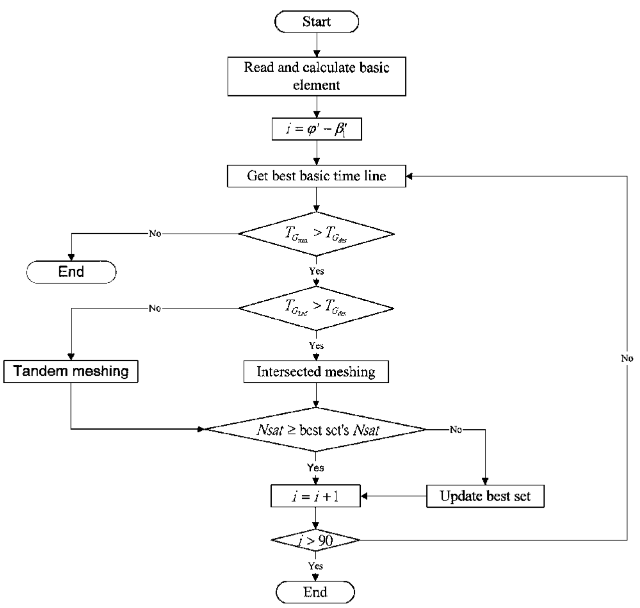

Fig. 6 Flowchart of constellation design.

Conclusions

In this study, the HET design procedure is modified and the method of constructing the time line is precisely designed. The criterion for choosing the best constellation is to minimize the number of satellites with the smallest value of inclination to meet the gap requirements. The new algorithm constructs the time lines exactly and then obtains the precise satellite constellations. The developed C++ program can find the most efficient arrangement of the satellite constellation and the optimal value of inclination in less than 10 min on a Pentium-based personal computer.

Acknowledgment

This work was sponsored by the National Science Council, Republic of China, under Contract NSC84-2612-E-032-001.

References

[1] Walker, J. G., "Circular Orbit Patterns Providing Whole Earth Coverage," Royal Aircraft Establishment, TR 70211, Nov. 1970.

[2] Draim, J. E., "A Common Period Four-Satellite Continuous Global Coverage Constellation," *Journal of Guidance, Control, and Dynamics*, Vol. 10, No. 5, 1987, pp. 492–499.

[3] Draim, J. E., "Continuous Global N-tuple Coverage with $2N + 2$ Satellites," American Astronomical Society, AAS Paper 89-381, Stowe, VT, Aug. 1989.

[4] Lang, T. J., "Symmetric Circular Orbit Satellite Constellations for Global Coverage," American Astronomical Society, AAS Paper 87-499, Kalispell, MT, Aug. 1987.

[5] Lang, T. J., "Optimal Low Earth Orbit Constellations for Continuous Global Coverage," American Astronomical Society, AAS Paper 93-597, Victoria, BC, Canada, Aug. 1993.

[6] Rider, L., "Analytic Design of Satellite Constellations for Zonal Earth Coverage Using Circular Orbits," *Journal of Astronautical Science*, Vol. 34, No. 1, 1986, pp. 31–64.

[7] Hanson, J. H., Evans, M. J., and Turner, R. E., "Designing Good Partial Coverage Satellite Constellations," *Journal of Astronautical Science*, Vol. 40, No. 2, 1992, pp. 215–239.

[8] Lang, T. J., and Hanson, J. M., "Orbital Constellations Which Minimize Revisit Time," American Astronomical Society, AAS Paper 83-402, Lake Placid, NY, Aug. 1983.

[9] Schmitt, D., "Low Altitude Satellite Constellations That Minimize Worldwide Revisit Time," American Astronomical Society, AAS Paper 89-379, Stowe, VT, Aug. 1989.

[10] Escobal, P. R., *Methods of Orbit Determination*, Krieger, Malabar, FL, 1976.

[11] Chylla, M. A., and Eagle, C. D., "Efficient Computation of Satellite Visibility Period," American Astronomical Society, AAS Paper 92-146, Colorado Springs, CO, Feb. 1992.

[12] Press, W. H., Flannery, B. P., Teukolsky, S. A., and Vetterling, W. T., *Numerical Recipes—The Art of Scientific Computing*, Cambridge Univ. Press, Cambridge, England, UK, 1986, Chap. 10.

F. H. Lutze Jr.
Associate Editor